# First Evidence for Center Dominance in SU(3) Lattice Gauge Theory

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**Abstract**

The dominance of center degrees of freedom is observed in SU(3) lattice gauge theory in maximal center gauge. The full asymptotic string tension is reproduced, after center projection, by the center elements alone. When center vortices are removed from lattice configurations, the string tension tends to zero. This provides further evidence for the role played by center vortices in the mechanism of color confinement in quantum chromodynamics, but more extensive simulations with a better gauge-fixing procedure are still needed.

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# 1 Introduction

The idea that center vortices play a decisive role in the mechanism of color confinement in quantum chromodynamics was proposed more than 20 years ago by 't Hooft [1] and other authors [2]. Recently, the center-vortex picture of confinement has found remarkable confirmation in numerical simulations of the SU(2) lattice gauge theory [3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Our group has proposed a technique for locating center vortices in thermalized lattice configurations based on fixing to the so called maximal center gauge, followed by center projection [3, 4, 5].

In SU(2) lattice gauge theory, the maximal center gauge is a gauge in which the quantity

$$R = \sum_x \sum_\mu \Big| \mathrm{Tr}[U_\mu(x)] \Big|^2 \tag{1}$$

reaches a maximum. This gauge condition forces each link variable to be as close as possible, on average, to a $Z_2$ center element, while preserving a residual $Z_2$ gauge invariance. Center projection is a mapping of each SU(2) link variable to the closest $Z_2$ center element:

$$U_\mu(x) \to Z_\mu(x) \equiv \mathrm{sign}\mathrm{Tr}[U_\mu(x)]. \tag{2}$$

The excitations on the projected $Z_2$ lattice are point-like, line-like, or surface-like objects, in $D = 2, 3$, or 4 dimensions respectively, called "P-vortices." These are thin objects, one lattice spacing across. There is substantial numerical evidence that thin P-vortices locate the middle of thick center vortices on the unprojected lattice. The string tension computed on center projected configurations reproduces the entire asymptotic SU(2) string tension [5]. It has also been demonstrated recently that removal of center vortices not only removes the asymptotic string tension, but restores chiral symmetry as well, and the SU(2) lattice is then brought to trivial topology [7]. The vortex density has been seen to scale as predicted by asymptotic freedom [8, 5, 6]. The properties of vortices have also been studied at finite temperature [9, 10, 11], and it has been argued that the non-vanishing string tension of spatial Wilson loops in the deconfined phase can be understood in terms of vortices winding through the periodic time direction. We have also proposed a simple model which explains the Casimir scaling of higher-representation string-tensions at intermediate distance scales in terms of the finite thickness of center vortices [13].

Putting the above and other pieces of evidence together, it seems clear that our procedure of maximal-center-gauge fixing and center projection identifies physical objects that play a crucial role in the mechanism of color confinement. However, the gauge group of QCD is color SU(3), not SU(2), and it is of utmost importance to demonstrate that the observed phenomena are not specific to the SU(2) gauge group only. Some preliminary results for SU(3) were presented in Section 5 of Ref. [5]. They came from simulations on very small lattices and at strong couplings. It was shown that center-projected Wilson loops reproduce results of the strong-coupling expansion of the full theory up to $\beta \simeq 4$.

The purpose of the present letter is to present further evidence on center dominance in SU(3) lattice gauge theory, very similar to the results that arose from SU(2) simulations. Though not as convincing as the SU(2) data, the first SU(3) results support the view that the vortex mechanism works in SU(3) in the same way as in SU(2).

## 2 Maximal Center Gauge in SU(3)

The maximal center gauge in SU(3) gauge theory is defined as the gauge which brings link variables $U$ as close as possible to elements of its center $Z_3$. This can be achieved as in SU(2) by maximizing a "mesonic" quantity

$$R = \sum_x \sum_\mu \mid \mathrm{Tr}\, U_\mu(x) \mid^2, \tag{3}$$

or, alternatively, a "baryonic" one

$$R' = \sum_x \sum_\mu \mathrm{Re} \left( \left[ \mathrm{Tr}\, U_\mu(x) \right]^3 \right). \tag{4}$$

The latter was the choice of Ref. [5], where we used the method of simulated annealing for iterative maximization procedure. The convergence to the maximum was rather slow and forced us to restrict simulations to small lattices and strong couplings.

The results, that will be presented below, were obtained in a gauge defined by the "mesonic" condition (3). The maximization procedure for this case is inspired by the Cabibbo–Marinari–Okawa SU(3) heat bath method [14, 15].[1] The idea of the method is as follows: In the maximization procedure we update link variables to locally maximize the quantity (3) with respect to a chosen link. At each site we thus need to find a gauge-transformation matrix $\Omega(x)$ which maximizes a local quantity

$$R(x) = \sum_\mu \left\{ \left| \mathrm{Tr} \left[ \Omega(x)\ U_\mu(x) \right] \right|^2 + \left| \mathrm{Tr} \left[ U_\mu(x-\hat{\mu})\, \Omega^\dagger(x) \right] \right|^2 \right\}. \tag{5}$$

Instead of trying to find the optimal matrix $\Omega(x)$, we take an SU(2) matrix $g(x)$ and embed it into one of the three diagonal SU(2) subgroups of SU(3). The expression (5) is then maximized with respect to $g$, with the constraint of $g$ being an SU(2) matrix. This reduces to an algebraic problem (plus a solution of a non-linear equation). Once we obtain the matrix $g(x)$, we update link variables touching the site $x$, and repeat the procedure for all three subgroups of SU(3) and for all lattice sites. This constitutes one center gauge fixing sweep. We made up to 1200 sweeps for each configuration. Center projection is then done by replacing the link matrix by the closest element of $Z_3$.

The above iterative procedure was independently developed by Montero, and described with full details in his recent publication [17]. Montero, building on the work of Ref. [18], has constructed classical SU(3) center vortex solutions on a periodic lattice. He has found that P-vortex plaquettes accurately locate the middle of the classical vortex, which is evidence of the ability of maximal center gauge to properly find vortex locations.

[1] A similar approach was applied for SU(3) cooling by Hoek et al. [16].

# 3 Center Dominance in SU(3) Lattice Gauge Theory

The effect of creating a center vortex linked to a given Wilson loop in SU(3) lattice gauge theory is to multiply the Wilson loop by an element of the gauge group center, i.e.

$$W(C) \to e^{\pm 2\pi i/3} W(C). \tag{6}$$

Quantum fluctuations in the number of vortices linked to a Wilson loop can be shown to lead to its area law falloff; the simplest, but urgent question is whether center disorder is sufficient to produce the whole asymptotic string tension of full, unprojected lattice configurations.

We have computed Wilson loops and Creutz ratios at various values of the coupling $\beta$ on a $12^4$ lattice, from full lattice configurations, center-projected link configurations in maximal center gauge, and also from configurations with all vortices removed. Figure 1 shows a typical plot at $\beta = 5.6$. It is obvious that center elements themselves produce a value of the string tension which is close to the asymptotic value of the full theory. On the other hand, if center elements are factored out from link matrices and Wilson loops are computed from SU(3)/$Z_3$ elements only, the Creutz ratios tend to zero for sufficiently large loops. The errorbars are, however, rather large, and one cannot draw an unambiguous conclusion from the data.

Recently we have argued that center dominance by itself does not prove the role of center degrees of freedom in QCD dynamics [20, 6]; some sort of center dominance exists also without any gauge fixing and can hardly by attributed to center vortices. Distinctive features of center-

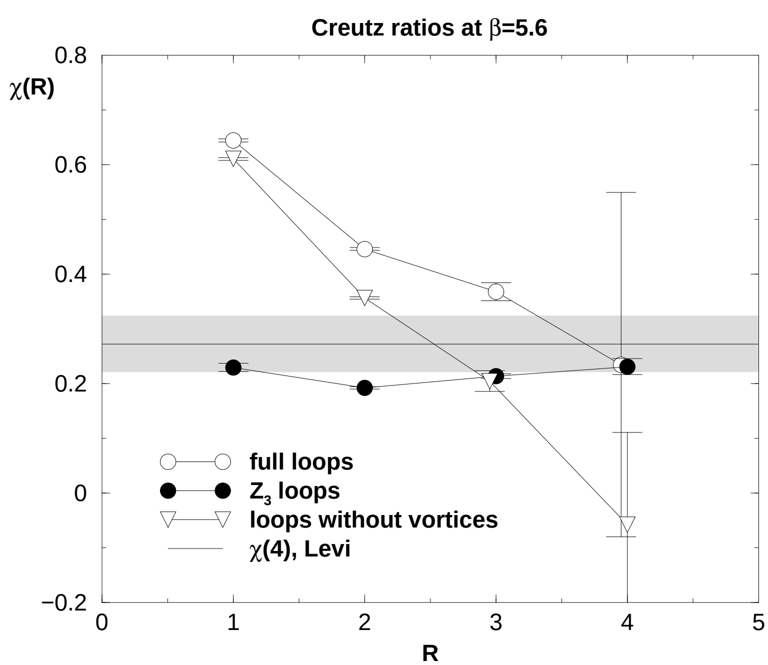


Figure 1: Creutz ratios for the original, the $Z_3$ projected, and the modified (with vortices removed) ensembles. ($\beta = 5.6$, $12^4$ lattice.) For comparison, the value of $\chi(4)$ and its error is shown in grey. The value comes from the compilation of Levi [19].

projected configurations in maximal center gauge in SU(2), besides center dominance, were that:
1. Creutz ratios were approximately constant starting from small distances (this we called "precocious linearity"),
2. the vortex density scaled with $\beta$ exactly as expected for a physical quantity with dimensions of inverse area.

Precocious linearity, the absence of the Coulomb part of the potential on the center-projected lattice at short distances, can be quite clearly seen from Fig. 1. One observes some decrease of the Creutz ratios at intermediate distances. A similar effect is present also at other values of $\beta$. It is not clear to us whether this decrease is of any physical relevance, or whether it should be attributed to imperfect fixing to the maximal center gauge.

The issue of scaling is addressed in Figure 2. Here values of various Creutz ratios are shown as a function of $\beta$ and compared to those quoted in Ref. [21]. All values for a given $\beta$ lie close to each other (precocious linearity once again) and are in reasonable agreement with asymptotic values obtained in time-consuming SU(3) pure gauge theory simulations. The plot in Fig. 2 is at the same time a hint that the P-vortex density also scales properly. The density is approximately proportional to the value of $\chi(1)$ in center-projected configurations, and $\chi(1)$ follows the same scaling curve as Creutz ratios obtained from larger Wilson loops.

A closer look at Fig. 2 reveals that there is no perfect scaling, similar to the SU(2) case, in our SU(3) data. Broken lines connecting the data points tend to bend at higher values of $\beta$. In our opinion, this is a finite-volume effect and should disappear for larger lattices. In our simulations we use the QCDF90 package [22] (supplemented by subroutines for MCG fixing and

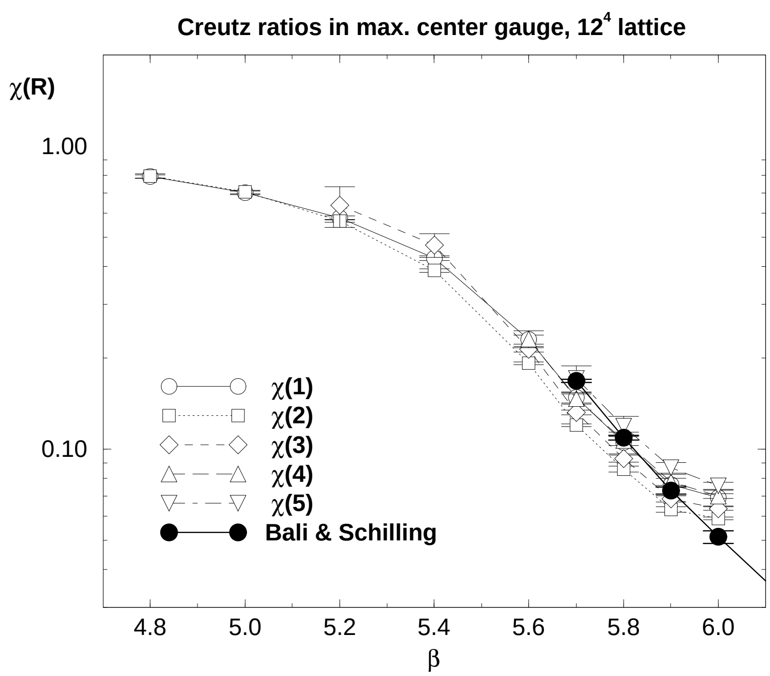


Figure 2: Center-projected Creutz ratios vs. $\beta$. Full circles connected with a solid line are asymptotic values quoted by Bali and Schilling [21].

center projection), which becomes rather inefficient on a larger lattice [23]. CPU and memory limitations do not allow us at present to extend simulations to larger lattice volumes.

An important test of the vortex-condensation picture is the measurement of vortex-limited Wilson loops. Let us denote $\langle W_n(C)\rangle$ the expectation value of the Wilson loop evaluated on a sub-ensemble of *unprojected* lattice configurations, selected such that precisely $n$ P-vortices, in the corresponding *center-projected* configurations, pierce the minimal area of the loop. For large loop areas one expects

$$\frac{\langle W_n(C)\rangle}{\langle W_0(C)\rangle} \to e^{2n\pi i/3}. \tag{7}$$

We tried to measure quantities like $\langle W_1(C)\rangle/\langle W_0(C)\rangle$ and $\langle W_2(C)\rangle/\langle W_0(C)\rangle$ in Monte Carlo simulations. The trend of our data is in accordance with the expectation based on the $Z_3$ vortex-condensation theory, Eq. (7), but before the evidence becomes conclusive, the errorbars become too large.

# 4 Conclusion

We have presented evidence for center dominance, precocious linearity, and scaling of center-projected Creutz ratios and of P-vortex density from simulations of the SU(3) lattice gauge theory. Our data – and conclusions that can be drawn from them – look quite similar to the case of SU(2). However, the SU(3) data at present are not as convincing and unambiguous as those of SU(2), the errorbars are still quite large and much more CPU time would be required to reduce them. The reason essentially is that the gauge-fixing maximization for SU(3) is very time consuming, either with simulated annealing or by the Cabibbo–Marinari–Okawa-like method used in the present investigation.[2] Moreover, the maximal center gauge is known to suffer from the Gribov problem, which makes gauge fixing notoriously difficult (in this context see also Refs. [24, 25]). A better alternative is badly needed, and may be provided by the recent proposal of de Forcrand et al. [26] based on fixing to the so-called Laplacian center gauge. Their first SU(2) results are promising, and the method can readily be extended to the case of SU(3).

It is encouraging that none of the pieces of data, which we have accumulated in SU(3) lattice gauge theory until now, contradicts conclusions drawn from earlier SU(2) results. If future extensive simulations with a more suitable, Gribov-copy free center-gauge fixing method confirm the evidence obtained in our exploratory investigation, center vortices will have a very strong claim to be the true mechanism of color confinement in QCD.

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[2] Typically thousands of iterations were needed for gauge fixing also in the investigation of Montero [17].

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